E.T.S.I.T.	**AMPLIACIÓN MATEMÁTICAS**
ECUACIONES EN DERIVADAS PARCIALES	PROBLEMAS – 2C

1.- Resolver los siguientes problemas con las condiciones que se indican:

(a) $\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}=0,\ \ 0<x<B;\ \ 0<y<A,\ \ u(0,y)=u(B,y)=u(x,A)=0;\ \ u(x,0)=f(x)$

(b) $\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}=0,\ \ 0<x<\pi;\ \ 0<y<A,\ \ u(0,y)=g(y);\ \ u(\pi,y)=u(x,0)=u(x,A)=0.$

(c) $\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}=0,\ \ 0<x<\pi;\ \ 0<y<\pi,$

$u(0,y)=u(\pi,y)=u(x,\pi)=0;\ \ u(x,0)=x^2(\pi-x)$

(d) $\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}=0,\ \ 0<x<\pi;\ \ 0<y<1,\ \ u(x,0)=u(x,1)=\operatorname{sen}^3 x;\ u(0,y)=\operatorname{sen}\pi y;$

$u(\pi,y)=0.$

2.- Resolver:

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0,\ \ 0<x<\pi;\ \ 0<y<\pi,\ \ u(x,0)=x^2;\ u(x,\pi)=0;\ \frac{\partial u}{\partial x}(0,y)=\frac{\partial u}{\partial x}(\pi,y)=0.$$

3.- Resolver:

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0,\ \ 0<x<1;\ \ 0<y<1,\ \ u(x,0)=(1-x)^2;\ u(x,1)=0;\ \frac{\partial u}{\partial x}(0,y)=\frac{\partial u}{\partial x}(1,y)=0.$$

<u>Soluciones</u>: Problema 1:

a) $u(x,y)=\displaystyle\sum_1^n b_n\frac{\operatorname{Sh}\left(n\pi(A-y)/B\right)}{\operatorname{Sh}\left(n\pi A/B\right)}\operatorname{sen}(n\pi x/B)$, con $b_n=\dfrac{2}{B}\displaystyle\int_0^B f(x)\operatorname{sen}(n\pi x/B)\,dx$.

b) $u(x,y)=\displaystyle\sum_1^n b_n\frac{\operatorname{Sh}\left(n\pi(\pi-x)/A\right)}{\operatorname{Sh}\left(n\pi^2/A\right)}\operatorname{sen}(n\pi y/A)$, con $b_n=\dfrac{2}{A}\displaystyle\int_0^A g(y)\operatorname{sen}(n\pi y/A)\,dy$.

c) $u(x,y)=-4\displaystyle\sum_1^n\left[1+2(-1)^n\right]n^{-3}\frac{\operatorname{Sh}\left(n(\pi-y)\right)}{\operatorname{Sh}(n\pi)}\operatorname{sen}(nx)$.

d) $u(x,y)=\dfrac{3\operatorname{Sh}(1-y)\operatorname{sen}x}{4\operatorname{Sh}1}-\dfrac{3\operatorname{Sh}3(1-y)\operatorname{sen}3x}{\operatorname{Sh}3}+\dfrac{3\operatorname{Sh}\pi(\pi-x)\operatorname{sen}\pi y}{\operatorname{Sh}\pi^2}$.

Problema 2: $u(x,y)=\dfrac{1}{3}\pi(\pi-y)+4\displaystyle\sum_1^n\frac{(-1)^n\operatorname{Sh}n(\pi-y)\cos nx}{n^2\operatorname{Sh}n\pi}$.

Problema 3:

$$u(x,y)=4\sum_1^n\left[\pi^{-2}\left(n-\tfrac{1}{2}\right)^{-2}+(-1)^n\pi^{-3}\left(n-\tfrac{1}{2}\right)^{-3}\right]\frac{\operatorname{Sh}\left(n-\frac{1}{2}\right)\pi(1-y)\cos\left(n-\frac{1}{2}\right)\pi x}{\operatorname{Sh}\left(n-\frac{1}{2}\right)\pi}.$$