

Proof Without Words: Sum of Triangular Numbers

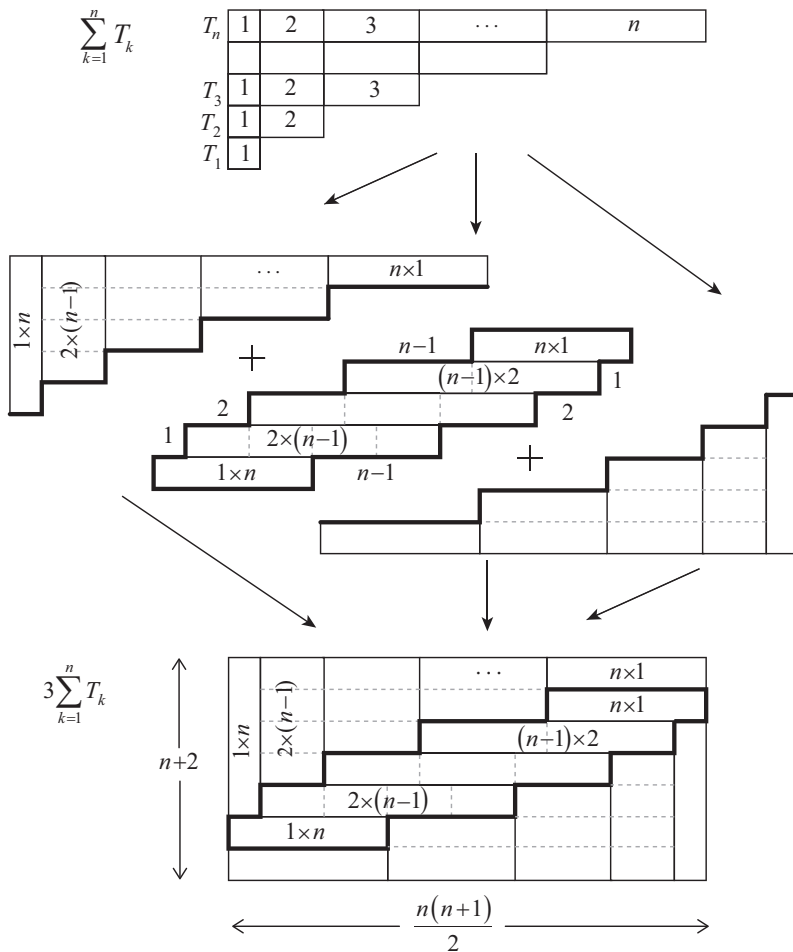
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Theorem.

$$T_n = 1 + 2 + \dots + n = \frac{n(n+1)}{2} = \binom{n+1}{2} \Rightarrow \sum_{k=1}^n T_k = \frac{n(n+1)(n+2)}{6}.$$

Proof.



This proof is close to, and it can be seen as a variation of, Zerger's proof [1], which also appears on page 94 of Nelsen's compendium of PWWs [2].

REFERENCES

1. M. J. Zerger, *Proof Without Words: Sum of Triangular Numbers*, *Math. Mag.* **63** no. 5 (1990) 314.
2. R. B. Nelsen, *Proof without Words: Exercises in Visual Thinking*. The Mathematical Association of America, Washington, DC, 1993.

Summary. The triangular numbers are given by the following explicit formulas: $T_n = 1 + 2 + \cdots + n = \frac{n(n+1)}{2} = \binom{n+1}{2}$. Here it is proved visually that

$$\sum_{k=1}^n T_k = \frac{n(n+1)(n+2)}{6}.$$

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From the Files of Past MAGAZINE Editors J. Arthur Seebach and Lynn Arthur Steen 1976–1980

As chairman of the MAA's Publications Committee, Ed Beckenbach asked then MAGAZINE co-editors Lynn Arthur Steen (LAS) and J. Arthur Seebach, Jr. (AS) what ideas they had for improving the MAGAZINE. They wanted to make it more public-oriented, but they realized they had no writers and no audience for such a magazine. Instead, they made some cosmetic changes, like putting something other than the table of contents on the cover, itself a controversial decision.

LAS and AS wanted to have articles start at the top of a page instead of simply starting where the previous article ended; ideally they would start at the top of a right-hand page. They needed some short things (called filler) to insert to fill space at the ends of some articles. At a suggestion of Roger Nelsen, they began including some Proofs without Words as filler. Proofs without Words have been a mainstay of the MAGAZINE ever since.