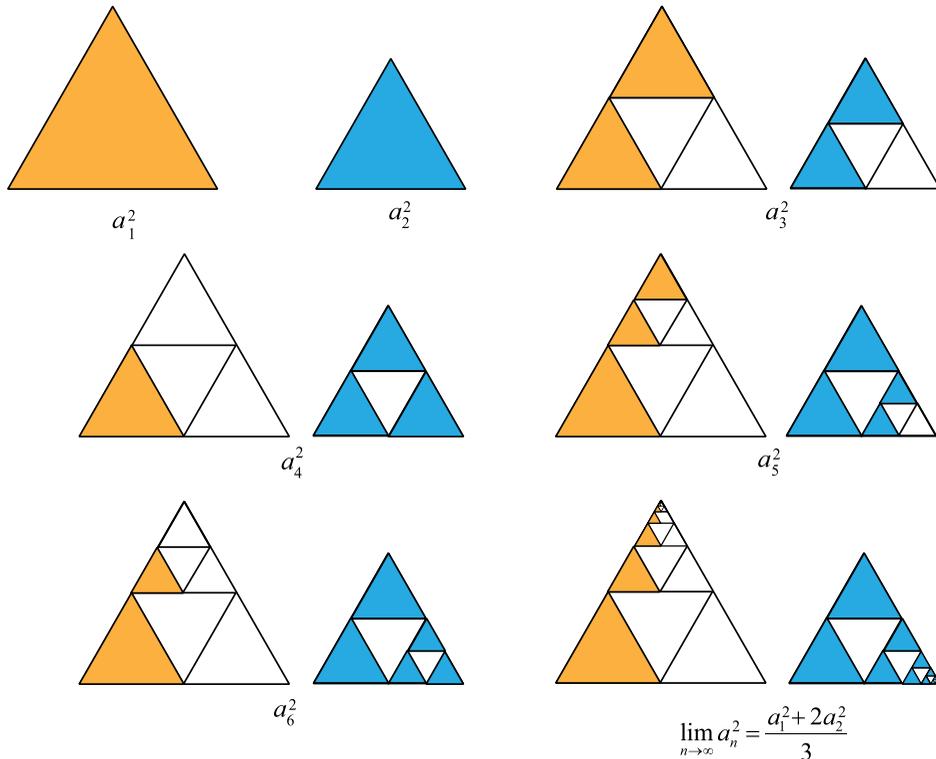


# Proof Without Words: Limit of a Recursive Root Mean Square

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Let  $(a_n)_{n \geq 1}$  be the sequence defined recursively by  $a_{n+1} = \sqrt{\frac{a_n^2 + a_{n-1}^2}{2}}$  for  $n \geq 2$ , with  $a_1$  and  $a_2$  two positive numbers as initial values. Then  $\lim_{n \rightarrow \infty} a_n = \sqrt{\frac{a_1^2 + 2a_2^2}{3}}$ .

*Proof.*



A related proof without words appears as [1].

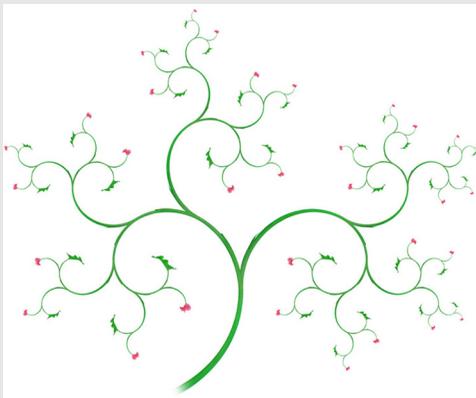
## REFERENCE

1. A. Plaza, Visual proof of the limit of f-mean recurrence sequences, *The Mathematical Gazette* **100** no. 576 (2016) 139–141.

**Summary.** Visual proof that the limit of the recursive root mean square sequence defined by  $a_{n+1} = \sqrt{\frac{a_n^2 + a_{n-1}^2}{2}}$  is  $\sqrt{\frac{a_1^2 + 2a_2^2}{3}}$  where  $a_1$  and  $a_2$  are the initial values of the sequence.

**ANGEL PLAZA** (MR Author ID: 350023) received his masters degree from Universidad Complutense de Madrid in 1984 and his Ph.D. from Universidad de Las Palmas de Gran Canaria in 1993, where he is a full professor in applied mathematics. He is interested in mesh generation and refinement, combinatorics, and visualization support in teaching and learning mathematics.

### Artist Spotlight Robert Fathauer



*Calla Hibiscus Spirals*, Robert Fathauer; limited edition screen print, 2012. This artwork was constructed by graphically iterating a photographic building block created from photographs of a calla lilly and a hibiscus flower, where the stalks of the calla lilly were distorted to conform to plastic-number spirals. In the print, a building block with a large leaf at one end and a blossom at the other was used at the beginning of the iteration process.

See interview on page 220.

### Ask Siri . . .

**A:** What is 1 divided by 0?

**S:** Please don't make me divide by 0. That would be like asking you to grow a third arm.

$$1 \div 0 = \text{undefined.}$$

Submitted by Anneliese Jones, Ann Arbor, MI