

Proof Without Words: Limit of a Recursive Arithmetic Mean

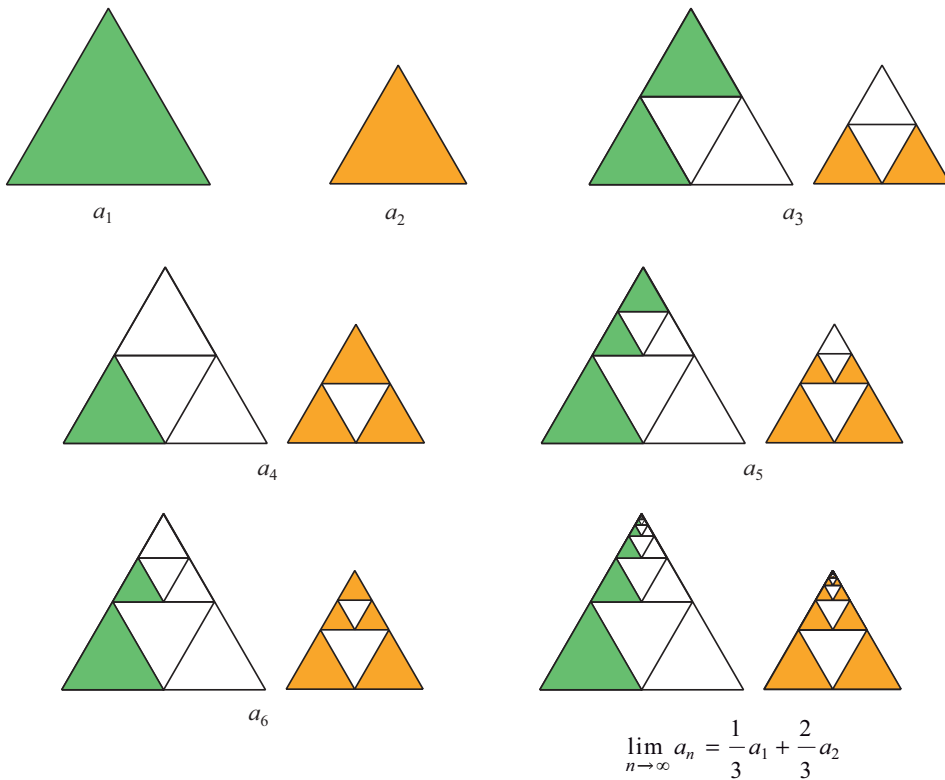
ÁNGEL PLAZA

Universidad de Las Palmas de Gran Canaria
Spain

angel.plaza@ulpgc.es

Let $(a_n)_{n \geq 1}$ be the sequence defined recursively by $a_{n+1} = \frac{a_n + a_{n-1}}{2}$ for $n \geq 2$, with a_1 and a_2 the initial values. Then $\lim_{n \rightarrow \infty} a_n = \frac{1}{3}a_1 + \frac{2}{3}a_2$.

Proof.



Summary. Visual proof that the limit of the recursive arithmetic mean sequence defined by $a_{n+1} = \frac{a_n + a_{n-1}}{2}$ is $\frac{a_1 + 2a_2}{3}$, where a_1 and a_2 are the initial values of the sequence.

ANGEL PLAZA (MR Author ID: [350023](#)) received his masters degree from Universidad Complutense de Madrid in 1984 and his Ph.D. from Universidad de Las Palmas de Gran Canaria in 1993, where he is a Full Professor in Applied Mathematics. He is interested in mesh generation and refinement, combinatorics and visualization support in teaching and learning mathematics.