Proof Without Words: Alternating Row Sums in Pascal's Triangle

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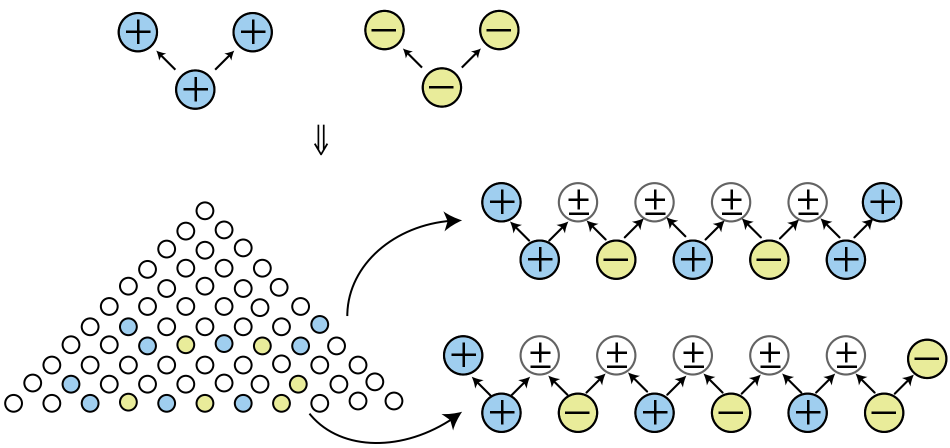
Theorem. For any integers $0 \le j \le m \le n$,

$$\sum_{k=j}^{m}(-1)^k\binom{n}{k} = (-1)^j\binom{n-1}{j-1} + (-1)^m\binom{n-1}{m},$$

and in particular if $j = 0$ and $m = n$, then $\sum_{k=0}^{n}(-1)^k\binom{n}{k} = 0$ (by defining as usual $\binom{n-1}{-1} = 0 = \binom{n-1}{n}$).

Proof. For simplicity we show the case when j is even; the odd cases can be obtained by reversing the role of $+$ and $-$.



$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k} \Rightarrow \sum_{k=j}^{m}(-1)^k\binom{n}{k} = (-1)^j\binom{n-1}{j-1} + (-1)^m\binom{n-1}{m}.$$

Summary. Based on the Pascal's identity, we visually demonstrate that the alternating sum of consecutive binomial coefficients in a row of Pascal's triangle is determined by two binomial coefficients from the previous row.

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