Proof Without Words: Partial Column Sums in Pascal's Triangle

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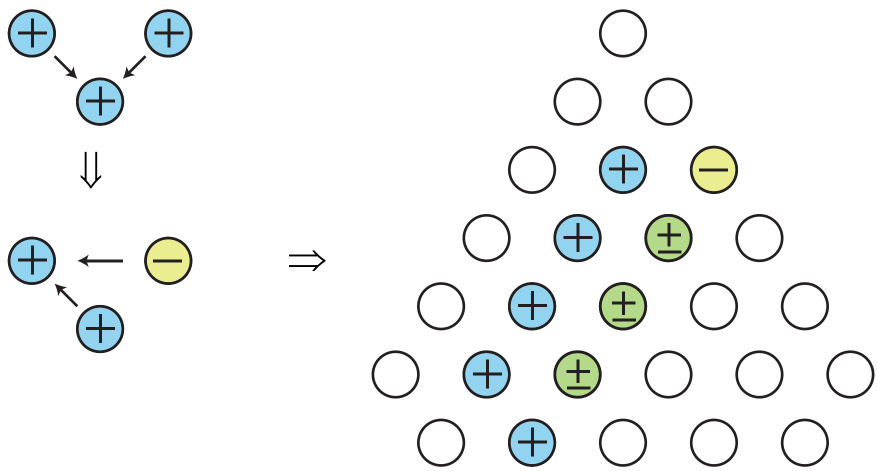
Theorem. For any integers $j \leq m \leq n$: $\sum_{k=m}^{n} \binom{k}{j} = \binom{n+1}{j+1} - \binom{m}{j+1}$.

Proof.



$$\binom{k}{j} = \binom{k+1}{j+1} - \binom{k}{j+1} \Rightarrow \sum_{k=m}^{n} \binom{k}{j} = \binom{n+1}{j+1} - \binom{m}{j+1}.$$

■

Exercise. Show that for any integers $j \geq 0$ and $m \leq n$:

$$\sum_{k=m}^{n} \binom{k+j}{k} = \binom{n+j+1}{n} - \binom{m+j}{m-1}.$$

Note: Setting $j = m$ and $\binom{m}{m+1} = 0$ in the theorem yields one form of the hockey stick identity, while setting $m = 0$ and $\binom{m+j}{-1} = 0$ in the exercise yields another form of the hockey stick identity. See [**1, 2**].

Math. Mag. **90** (2017) 117–118. doi:10.4169/math.mag.90.2.117. © Mathematical Association of America
MSC: Primary 05A10.

REFERENCES

1. P. Hilton, J. Pedersen, Looking into Pascal's triangle: Combinatorics, arithmetic, and geometry, *Math. Mag.* **60** no. 5 (1987) 305–316.
2. S. Mehri, J. Pedersen, The hockey stick theorems in Pascal and trinomial triangles, arXiv:1404.5106v1. [math.HO] 21 Apr. 2014.

Summary. Based on the binomial property $\binom{k+1}{j+1} = \binom{k}{j} + \binom{k}{j+1}$, written as $\binom{k}{j} = \binom{k+1}{j+1} - \binom{k}{j+1}$, the sum of consecutive column entries of Pascal's triangle is written as a difference of two binomial coefficients in the next column, which generalizes the so-called hockey stick identities.

ÁNGEL PLAZA (MR Author ID: 350023) received his masters degree from Universidad Complutense de Madrid in 1984 and his Ph.D. from Universidad de Las Palmas de Gran Canaria in 1993, where he is a full professor in applied mathematics. He is interested in mesh generation and refinement, combinatorics and visualization support in teaching and learning mathematics.

PINEMI PUZZLE

			8		5		4	4	
	16			6		7	7		3
			12	11	7	8		8	4
6	9	8							
	4		10			9	9		8
4		7		8	10		12	11	
	7		6		9				9
6		5	6				11		
	7		6		10		8		
4		5				7		6	3

How to play. Place one jamb (|), two jambs (||), or three jambs (|||) in each empty cell. The numbers indicate how many jambs there are in the surrounding cells—including diagonally adjacent cells. Each row and each column has 10 jambs. Note that no jambs can be placed in any cell that contains a number.

The solution is on page 141.

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