Proof Without Words: Mengoli's Series

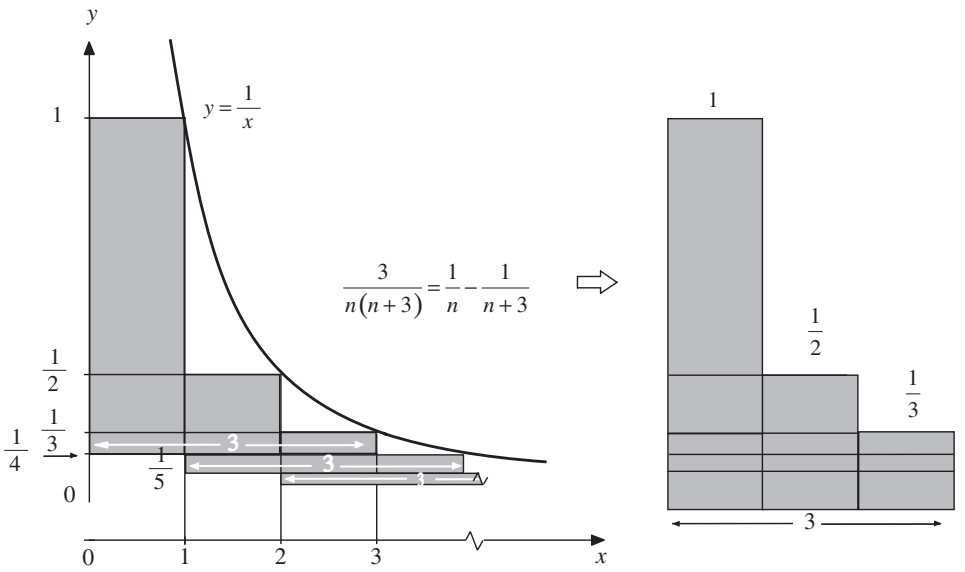
Pietro Mengoli posed the problem of summing the series below and found the sum for the cases $1 \le k \le 10$. [**1**]

$$\sum_{n=1}^{\infty} \frac{k}{n(n+k)} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{k}$$

We illustrate the case $k = 3$.

$$\frac{3}{n(n+3)} = \frac{1}{n} - \frac{1}{n+3} \Rightarrow$$

$$\sum_{n=1}^{\infty} \frac{3}{n(n+3)} = 1 - \cancel{\frac{1}{4}} + \frac{1}{2} - \cancel{\frac{1}{5}} + \frac{1}{3} - \cancel{\frac{1}{6}} + \cancel{\frac{1}{4}} - \cdots = 1 + \frac{1}{2} + \frac{1}{3}$$



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REFERENCE

1. Pietro Mengoli, *Novae quadraturae arithmeticae, seu de additione fractionum*. Bologna, 1650.