Consider the set, S of polynomials in one variable over the integers with zero coefficient on the linear term. That is to say consider:

$$S = \left\{a_n \cdot x^n + a_{n-1} \cdot x^{n-1} + a_{n-2} \cdot x^{n-2} + \cdots + a_2 \cdot x^2 + a_0\right\}$$

Now it's easy to verify that S is an integral domain. But the delightful surprise is that this integral domain does *not* have unique factorization into *irreducibles* (nonunit elements x such that if $x = yz$ then y or z is a unit) and that this is clear immediately from what follows. Consider x^6. This can be written as $x^2 \cdot x^2 \cdot x^2$ or $x^3 \cdot x^3$. Both x^2 and x^3 are clearly irreducible in S. And since these two factorizations into irreducibles contain different numbers of factors, they are distinct.

As a bonus we also find that x^2 and x^3 are not *prime* illustrating that *prime* and *irreducible* are separate concepts. For example, x^2 divides $x^3 \cdot x^3$ while not dividing either factor.

REFERENCE

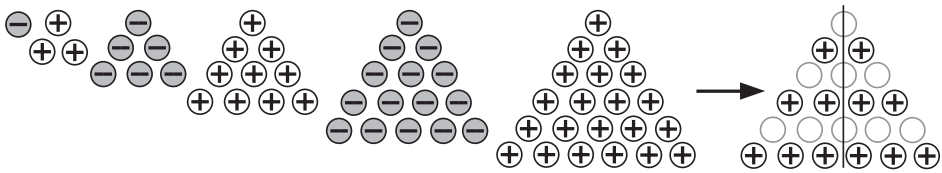
1. Joseph J. Rotman, *Advanced Modern Algebra*, Pearson, 2002, p. 330.

Proof Without Words: Alternating Sum of an Even Number of Triangular Numbers

ÁNGEL PLAZA
ULPGC
35017-Las Palmas G.C., Spain
aplaza@dmat.ulpgc.es

$$t_k = 1 + 2 + \ldots + k \;\Rightarrow \sum_{k=1}^{2n} (-1)^k t_k = 2t_n$$

E.g., $n = 3$:



$$-t_1 + t_2 \quad -t_3 \quad +t_4 \quad -t_5 \quad +t_6 \quad = \quad 2t_3$$

NOTE. For a "proof without words" of a similar statement—alternating sums of an odd number of triangular numbers—see Roger B. Nelsen, this MAGAZINE, Vol. 64, no. 4 (1995), p. 284.