

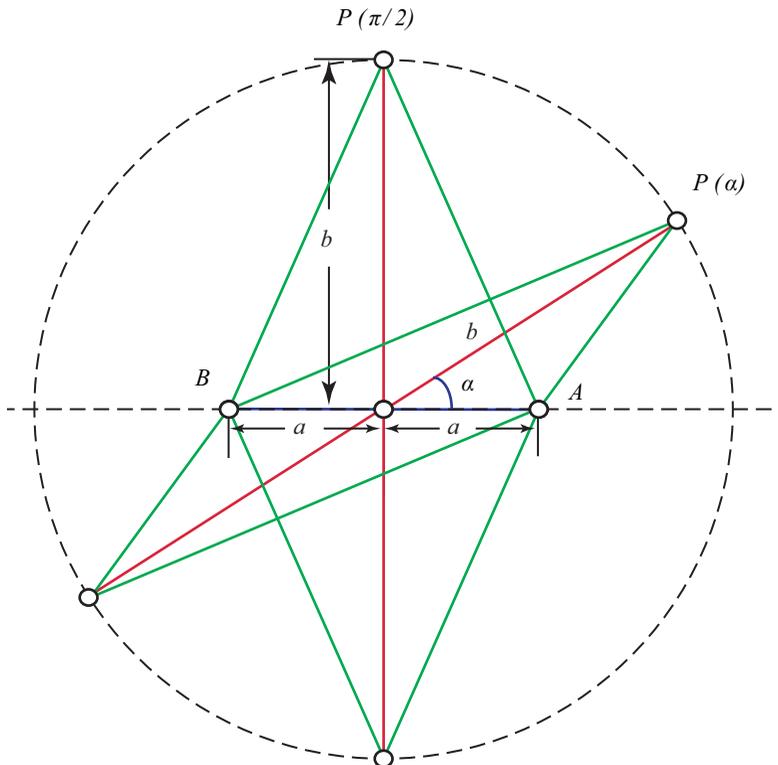
The Parallelogram with Maximum Perimeter for Given Diagonals Is the Rhombus—A Proof Without Words and a Corollary

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Theorem. *The parallelogram with maximum perimeter for given diagonals is the rhombus.*

Proof.

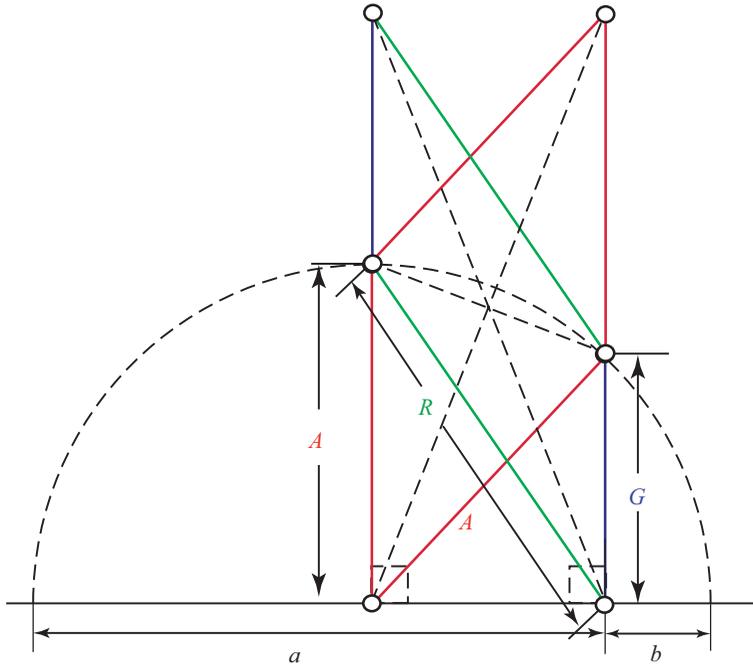


$$\begin{aligned}
 |AP(\alpha)| + |BP(\alpha)| &= \sqrt{a^2 + b^2 - 2ab \cos \alpha} + \sqrt{a^2 + b^2 + 2ab \cos \alpha} \\
 &\leq 2\sqrt{a^2 + b^2}
 \end{aligned}$$

(by the Law of Cosines and the arithmetic mean-root mean square inequality).

Corollary. *For two positive numbers a and b , let us denote by A , G , and R , respectively their arithmetic mean, geometric mean, and root mean square, then $2A \geq R + G$.*

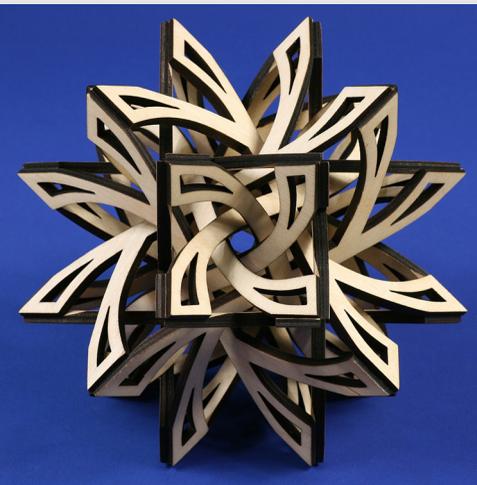
Proof.



(where two parallelograms with equal diagonals have been constructed from the geometric representation of the arithmetic mean, geometric mean, and root mean square of two positive given numbers).

Summary. By the Law of Cosines and the arithmetic mean-root mean square inequality it is proved without words that The Parallelogram with Maximum Perimeter for given Diagonals is the Rhombus. As a corollary it also proved that for two positive numbers, their arithmetic mean is greater or equal than the arithmetic mean of their geometric mean and their root mean square.

ANGEL PLAZA (MR Author ID: [350023](#)) received his masters degree from Universidad Complutense de Madrid in 1984 and his Ph.D. from Universidad de Las Palmas de Gran Canaria in 1993, where he is a Full Professor in Applied Mathematics. He is interested in mesh generation and refinement, combinatorics and visualization support in teaching and learning mathematics.



Artist Spotlight George Hart

Dragonflies, George Hart; wood, 7.5 inches in diameter, 2009; private collection. Constructed from 12 identical plywood components.

See interview on page 374.