Proof Without Words: The Triangle with Maximum Area for a Given Base and Perimeter

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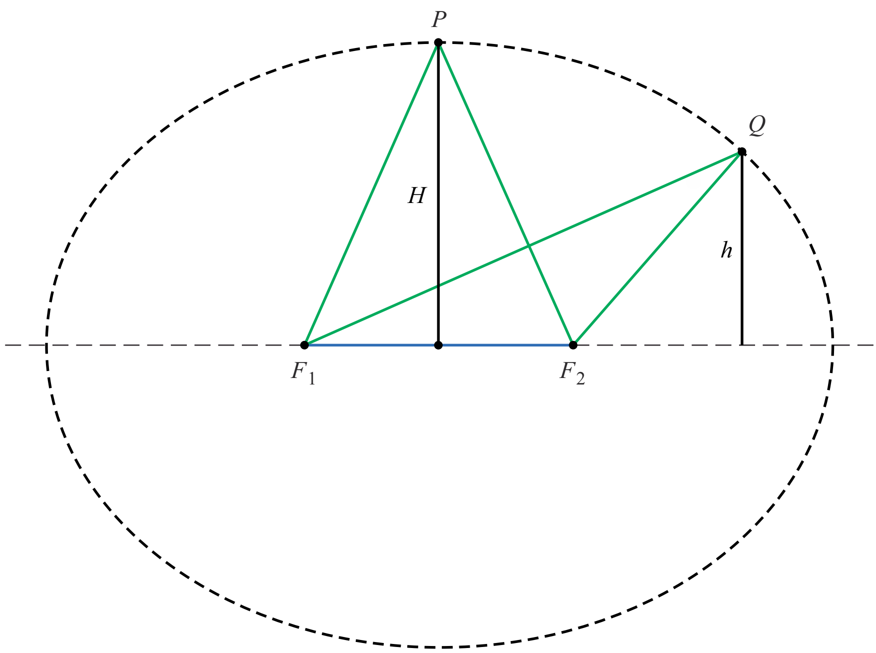
Theorem. *The triangle with maximum area for a given base and perimeter is the isosceles triangle where the different edge is the base.*

Proof.



$$|F_1P| + |F_2P| = |F_1Q| + |F_2Q| .$$

$$H \geq h \Longrightarrow \text{Area}(F_1PF_2) \geq \text{Area}(F_1QF_2).$$ ■

Corollary (Isoperimetric theorem for triangles). *The triangle with maximum area for a given perimeter is the equilateral triangle.*

Proof hint. Apply the theorem iteratively, choosing the middle length edge as the base. In the limit, the equilateral triangle is obtained.

Summary. By using the ellipse with foci at the extreme points of the base, we show wordlessly that the triangle with maximum area for a given base and perimeter is the isosceles triangle where the different edge is the base.

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