Hence,

$$\begin{aligned} R = \frac{221}{2},\ \cos\alpha &= \frac{140}{221},\ \sin\alpha = \frac{171}{221} \\ \cos\beta &= \frac{104}{221},\ \sin\beta = \frac{195}{221} \\ \cos\gamma &= \frac{85}{221},\ \sin\gamma = \frac{204}{221}, \end{aligned}$$

and our result for the perimeter of $\triangle ACE$.

It is easy to check that $\angle BFD = \alpha, \angle FDB = \beta, \angle DBF = \gamma$ so that $\angle BAF = \pi - \beta, \angle DEF = \pi - \gamma$.

Applying the cosine formula to $\triangle BAF$ and $\triangle DEF$ respectively, we obtain $BF = 195$ and $DF = 204$.

It follows, as claimed, that the area of

$$\triangle BDF = \frac{1}{2}\left(\overline{BF}\right)\left(\overline{DF}\right)\sin\angle BFD = \frac{1}{2}(195)(204)\frac{171}{221} = 15390.$$

Also solved by Bruno Salgueiro Fanego, Viveiro, Spain; David E. Manes, Oneonta, NY; Boris Rays, Chesapeake, VA; David Stone and John Hawkins (jointly), Statesboro, GA, and the proposer.

- **5046:** *Proposed by R.M. Welukar of Nashik, India and K.S. Bhanu, and M.N. Deshpande of Nagpur, India.*

  Let $4n$ successive Lucas numbers $L_k, L_{k+1}, \cdots, L_{k+4n-1}$ be arranged in a $2 \times 2n$ matrix as shown below:

$$\begin{array}{cccccc} 1 & 2 & 3 & 4 & \cdots & 2n \end{array}$$
$$\begin{pmatrix} L_k & L_{k+3} & L_{k+4} & L_{k+7} & \cdots & L_{k+4n-1} \\ L_{k+1} & L_{k+2} & L_{k+5} & L_{k+6} & \cdots & L_{k+4n-2} \end{pmatrix}$$

Show that the sum of the elements of the first and second row denoted by R_1 and R_2 respectively can be expressed as

$$R_1 = 2F_{2n}L_{2n+k}$$

$$R_2 = F_{2n}L_{2n+k+1}$$

where $\{L_n, n \geq 1\}$ denotes the Lucas sequence with $L_1 = 1, L_2 = 3$ and $L_{i+2} = L_i + L_{i+1}$ for $i \geq 1$ and $\{F_n, n \geq 1\}$ denotes the Fibonacci sequence, $F_1 = 1, F_2 = 1, F_{n+2} = F_n + F_{n+1}$.

Solution by Angel Plaza and Sergio Falcon, Las Palmas, Gran Canaria, Spain.

$R_1 = L_k + L_{k+3} + L_{k+4} + L_{k+7} + \cdots + L_{k+4n-2} + L_{k+4n-1}$, and since $L_i = F_{i-1} + F_{i+1}$, we have:

$$
\begin{aligned}
R_1 &= F_{k-1}+F_{k+1}+F_{k+2}+F_{k+4}+F_{k+3}+F_{k+5}+\cdots+F_{k+4n-2}+F_{k+4n}\\
&= F_{k-1}+\sum_{j=1}^{4n}F_{k+j}-F_{k+4n-1}\\
&= F_{k-1}-F_{k+4n-1}+\sum_{j=0}^{4n+k}F_j-\sum_{j=0}^{k}F_j
\end{aligned}
$$

And since $\sum_{j=0}^{m}F_j=F_{m+2}-1$ we have:

$$
R_1 = F_{k-1}-F_{k+4n-1}+F_{k+4n+2}-1-F_{k+2}+1=2F_{k+4n}-2F_k
$$

where in the last equation it has been used that $F_{i+2}-F_i=F_{i+1}+F_i-F_{i-1}=2F_i$.
Now, using the relation $L_nF_m=F_{n+m}-(-1)^mF_{n-m}$ (S. Vajda, Fibonacci and Lucas numbers, and the Golden Section: Theory and Applications, Dover Press (2008)) in the form $L_{2n+k}F_{2n}=F_{4n+k}-(-1)^{2n}F_{2n+k-2n}$ it is deduced $R_1=2F_{2n}L_{2n+k}$.
In order to prove the fomula for R_2 note that

$$
R_1+R_2=\sum_{j=0}^{4n-1}L_{k+j}=\sum_{j=0}^{4n+k-1}L_j-\sum_{j=0}^{k-1}L_j
$$

As before, $\sum_{j=0}^{4n+k-1}L_j=F_{k+4n}+F_{k+4n+2}$, while $\sum_{j=0}^{k-1}L_j=F_k+F_{k+2}$, so

$$
\begin{aligned}
R_1+R_2 &= F_{k+4n}-F_k+F_{k+4n+2}-F_{k+2}\\
&= L_{2n+k}F_{2n}+L_{2n+k+2}F_{2n}
\end{aligned}
$$

And therefore,

$$
R_2 = F_{2n}\left(L_{2n+k+2}-L_{2n+k}\right)=F_{2n}L_{2n+k+1}
$$

Also solved by Paul M. Harms, North Newton, KS; John Hawkins and David Stone (jointly), Statesboro, GA, and the proposers.)

- 5047: *Proposed by David C. Wilson, Winston-Salem, N.C.*

Find a procedure for continuing the following pattern:

$$
\begin{aligned}
S(n,0) &= \sum_{k=0}^{n}\binom{n}{k}=2^n\\
S(n,1) &= \sum_{k=0}^{n}\binom{n}{k}k=2^{n-1}n\\
S(n,2) &= \sum_{k=0}^{n}\binom{n}{k}k^2=2^{n-2}n(n+1)
\end{aligned}
$$