 # 5060 Proposed by José Luis Díaz-Barrero, Barcelona, Spain.

Show that there exists $c \in (0, \pi/2)$ sucht that

$$\int_0^c \sqrt{\sin x}\ dx + c\sqrt{\cos c} = \int_c^{\pi/2} \sqrt{\cos x}\ dx + (\pi/2 - c)\sqrt{\sin c}$$

Solution: by Angel Plaza, University of Las Palmas de Gran Canaria, 35017-Las Palmas G.C., Spain.

Consider the continuos function $F(x)$ defined on $[0, \pi/2]$ as follows:

$$F(x) = \int_0^x \sqrt{\sin t}\ dt + x\sqrt{\cos x} - \int_x^{\pi/2} \sqrt{\cos t}\ dt + (\pi/2 - x)\sqrt{\sin x}$$

Note that:

$$\begin{aligned} F(0) &= -\int_0^{\pi/2} \sqrt{\cos t}\ dt < 0 \\ F(\pi/2) &= \int_0^{\pi/2} \sqrt{\sin t}\ dt > 0 \end{aligned}$$

Hence, by the Bolzano's Theorem there exist $c \in (0, \pi/2)$ sucht that $F(c) = 0$ and the given equality is done. □