Solution to Problem # B-1051

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Problem# B-1051 *Proposed by Charles K. Cook, Sumter, SC*

For all positive integers n show that $F_n^4 + L_n^4 - 6F_n^2 + 5 > 0$.

Solution. *by Sergio Falcón & Ángel Plaza, ULPGC, Spain*

Let us call $A(n) = F_n^4 + L_n^4 - 6F_{2n}$.

Since $F_n = F_{n+1} - F_{n-1}$, $L_n = F_{n+1} + F_{n-1}$, and $F_{2n} = (F_{n+1} - F_{n-1})F_n$, then

$$\begin{aligned} A(n) &= (F_{n+1} - F_{n-1})^4 + (F_{n+1} + F_{n-1})^4 - 6(F_{n+1}^2 - F_{n-1}^2) \\ &= 2F_{n+1}^4 + 12F_{n+1}^2F_{n-1}^2 + 2F_{n-1}^4 - 6F_{n+1}^2 + 6F_{n-1}^2 \\ &= 2F_{n+1}^2(F_{n+1}^2 - 3) + 2F_{n+1}^2(F_{n-1}^2 + 6F_{n+1}^2 + 3) \end{aligned}$$

Note that for $n = 1$, $A(1) + 5 = 2 \cdot 1(1 - 3) + 5 = 1 > 0$. For $n \geq 2$, since $F_{n+1} \geq 2$, then $A(n) > 1$, and therefore $A(n) + 5 > 0$. $\square$

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