# Solution to Problem # B-1058

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**Problem# B-1058** *Proposed by M. N. Despande, Nagpur, India*

Prove the following identities:

(1) $$9(F_{n+1}^4 + F_n^4 + F_{n-1}^4) - (F_{n+2}^4 + F_{n-2}^4) = L_n^4$$

(2) $$9(L_{n+1}^4 + L_n^4 + L_{n-1}^4) - (L_{n+2}^4 + L_{n-2}^4) = 625F_n^4$$

**Solution.** The identities may be proved by using Problem B-1044:

(3) $$L_n^2 = 2F_{n+1}^2 - F_n^2 + 2F_{n-1}^2;$$

(4) $$25F_n^2 = 2L_{n+1}^2 - L_n^2 + 2L_{n-1}^2$$

Taking squares in (3) and using that $F_{n+1} = F_n + F_{n-1}$ it is obtained

$$L_n^4 = (2F_{n+1}^2 - F_n^2 + 2F_{n-1}^2)^2 = F_n^4 + 16F_{n-1}^4 + 32F_nF_{n-1}^3 + 8F_n^3F_{n-1} + 24F_n^2F_{n-1}^2$$

The same result is obtained from the left-hand side of (1), using now also that $F_{n-2} = F_n - F_{n-1}$.

The proof of (2) follows analogously from (4), and using now that $L_{n+1} = L_n + L_{n-1}$ and $L_{n-2} = L_n - L_{n-1}$. □

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