**PROBLEM OF THE WEEK. Problem No. 7 (Fall 2010 Series)**

Let $n \geq 2$ and let $0 < x_i < 1$, $i = 1, 2, \ldots, n$. Show that $\prod_{i=1}^{n}(1 - x_i) + \sum_{i=1}^{n} x_i > 1$.

**Solution by Angel Plaza, ULPGC, Spain**

By induction. For $n = 2$: $0 < x_1, x_2 < 1$, it is obtained

$(1 - x_1)(1 - x_2) + x_1 + x_2 = 1 - x_1 - x_2 + x_1x_2 + x_1 + x_2 = 1 + x_1x_2 > 1.$

Let us suppose that the inequality holds for $0 < x_i < 1$, $i = 1, 2, \ldots, n - 1$. The inequality for $n$ numbers follows immediately by writing

$$\prod_{i=1}^{n}(1 - x_i) + \sum_{i=1}^{n} x_i = 1 + \underbrace{\left(\prod_{i=1}^{n-1}(1 - x_i) + \sum_{i=1}^{n-1} x_i - 1\right)}_{>0} \underbrace{(1 - x_n)}_{>0} + \underbrace{x_n \sum_{i=1}^{n-1} x_i}_{>0} > 1.$$