11343. Proposed by David Beckwith, Sag Harbor, NY.

Show that when n is a positive integer,

$$\sum_{k\geq 0}\binom{n}{k}\binom{2k}{k}=\sum_{k\geq 0}\binom{n}{2k}\binom{2k}{k}3^{n-2k}$$

Solution: *(by Ángel Plaza, ULPGC, 35017-Las Palmas G.C., Spain, e-mail: aplaza@dmat.ulpgc.es)*

Since $1+\left(x+\frac{1}{x}\right)^2=3+x^2+\frac{1}{x^2}$, the result follows by considering the constant term in each term of the equation $\left(1+\left(x+\frac{1}{x}\right)^2\right)^n=\left(3+\left(x^2+\frac{1}{x^2}\right)\right)^n$:

$$\left(1+\left(x+\tfrac{1}{x}\right)^2\right)^n=\sum_{k\geq 0}\binom{n}{k}\left(\left(x+\frac{1}{x}\right)^2\right)^k=\sum_{k\geq 0}\binom{n}{k}\left(x+\frac{1}{x}\right)^{2k},$$

where the constant term is $\displaystyle\sum_{k\geq 0}\binom{n}{k}\binom{2k}{k}$.

On the other hand,

$\left(3+\left(x^2+\frac{1}{x^2}\right)\right)^n=\displaystyle\sum_{k\geq 0}\binom{n}{k}3^{n-k}\left(x^2+\frac{1}{x^2}\right)^k$, where the constant term is $\displaystyle\sum_{k\geq 0}\binom{n}{2k}3^{n-2k}\binom{2k}{k}$. $\square$