898. Proposed by Mihály Bencze, Sacele, Brasov, Romania.

Suppose f is a function defined on an open interval I such that $f''(x) \geq 0$ for all $x \in I$, and $[a, b] \subset I$. Prove that

$$\int_0^1 f(a + (b - a)y)dy \geq \int_0^1 f\left(\frac{3a + b}{4} + \frac{b - a}{2}y\right) dy$$

Solution: (by Ángel Plaza, and Sergio Falcón, University of Las Palmas de Gran Canaria, 35017-Las Palmas G.C., Spain)

By means of a suitable changes of variables in the integrals the proposed inequality may be written as:

$$\frac{1}{b-a}\int_a^b f(x)dx \geq \frac{1}{b-a}\int_a^b f\left(\frac{x}{2} + \frac{a+b}{4}\right) dx \tag{1}$$

Since $f''(x) \geq 0$ for all $x \in I$, f is convex and then

$$f\left(\frac{x}{2} + \frac{a+b}{4}\right) \leq \frac{1}{2}f(x) + \frac{1}{2}f\left(\frac{a+b}{2}\right)$$

By integrating previous inequelity we obtain:

$$\begin{aligned}
\frac{1}{b-a}\int_a^b f\left(\frac{x}{2} + \frac{a+b}{4}\right) dx &\leq \frac{1/2}{b-a}\int_a^b f(x)dx + \frac{1}{2}f\left(\frac{a+b}{2}\right) \\
&\leq \frac{1/2}{b-a}\int_a^b f(x)dx + \frac{1/2}{b-a}\int_a^b f(x)dx \\
&= \frac{1}{b-a}\int_a^b f(x)dx
\end{aligned}$$

From where Equation (1) follows. $\square$