883. Proposed by Brian Bradie, Christopher Newport University, Newport News, Virginia.

Evaluate

$$(a) \int_0^1 \frac{\ln(1+x)}{1+x^2} dx,$$

$$(b) \int_0^1 \frac{\arctan x}{1+x} dx.$$

Solution: *(by Ángel Plaza and Sergio Falcón, University of Las Palmas de Gran Canaria, 35017-Las Palmas G.C., Spain)*

The required computation in (a), $\int_0^1 \frac{\ln(1+x)}{1+x^2} dx$, appears in various tables and its evaluation was given as Problem A5 on the Sixty-Sixth William Lowell Putnam Competition (see The American Mathematical Monthly 113 (2006) 733-743, and also problem 11277 in the Monthly 115-8 (2008) 758-759):

$$\int_0^1 \frac{\ln(1+x)}{1+x^2} dx = \frac{\pi \ln 2}{8},$$

Now the evaluation of integral in (b) follows straightforwardly from the fact that $(\ln(1+x)\arctan x)' = \frac{\arctan x}{1+x} + \frac{\ln(1+x)}{1+x^2}$, and therefore

$$\begin{aligned} \int_0^1 \frac{\arctan x}{1+x} dx &= [\ln(1+x)\arctan x]_0^1 - \int_0^1 \frac{\ln(1+x)}{1+x^2} dx = \\ &= \frac{\pi \ln 2}{4} - \frac{\pi \ln 2}{8} = \frac{\pi \ln 2}{8} \end{aligned}$$

$\square$