Solution to MM Problem # 1826

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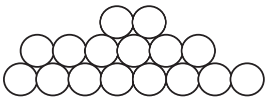
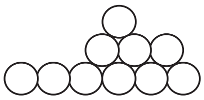
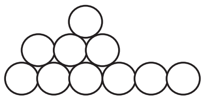
1826. *Proposed by Michael Woltermann, Washington & Jefferson College, Washington, PA.*

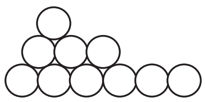
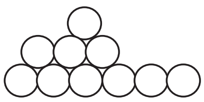
A block fountain of coins is an arrangement of n identical coins in rows such that the coins in the first row form a contiguous block, and each row above that forms a contiguous block. As an example,



If a_n denotes the number of block fountains with exactly n coins in the base, then $a_n = F_{2n-1}$, where F_k denotes the kth Fibonacci number. (Wilf, generatingfunctionology, 1994.) How many block fountains are there if two fountains that are mirror images of each other are considered to be the same? That is, if two fountains such as

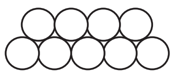
 

are the same, while two fountains such as

are different?

Solution. Let us denote by S_n the number of block fountains with exactly n coins in the base, and such the mirror image of each one coincides with itself. As an example,



Then, $a_n = S_n + \dfrac{F_{2n-1} - S_n}{2} = \dfrac{F_{2n-1} + S_n}{2}$.

In order to find S_n note, that $S_1 = 1$, $S_{2n} = 1 + \sum_{k=1}^{n} S_{2k-1}$, and $S_{2n+1} = 1 + \sum_{k=1}^{n} S_{2k}$. This implies that $S_1 = 1$, $S_2 = 2$, and $S_{n+2} = S_{n+1} + S_n$, for $n \geq 1$. Hence, $S_n = F_{n+1}$. Therefore, $a_n = \dfrac{F_{2n-1} + F_{n+1}}{2}$. $\square$