

**Problem No. 2002.** (*Proposed by Dorin Marghidanu, Colegiul National "A. I. Cuza," Corabia, Romania.*)

Let  $a_1, a_2, \dots, a_n$  be positive real numbers. Prove that

$$\frac{a_1^2}{a_1 + a_2} + \frac{a_2^2}{a_1 + a_2} + \cdots + \frac{a_{n-1}^2}{a_{n-1} + a_n} + \frac{a_n^2}{a_n + a_1} \geq \frac{a_1 + a_2 + \cdots + a_n}{2}$$

**Solution by Ángel Plaza, University of Las Palmas de Gran Canaria, 35017-Las Palmas G.C., Spain**

The inequality is a straightforward consequence of the Bergström's inequality (see H. Bergström, *A triangle inequality for matrices*, Den Elfte Skandinaviske Matematikerkongress, Trondheim, 1949, Johan Grundt Tanums Forlag, Oslo, 1952, 264-267.). Namely:

If  $x_k \in R$ ,  $a_k > 0$ ,  $(1 \leq k \leq n)$ , then

$$\frac{x_1^2}{a_1} + \frac{x_2^2}{a_2} + \cdots + \frac{x_n^2}{a_n} \geq \frac{(x_1 + x_2 + \cdots + x_n)^2}{a_1 + a_2 + \cdots + a_n}$$

where the equality holds when  $\frac{x_1^2}{a_1} = \frac{x_2^2}{a_2} = \cdots = \frac{x_n^2}{a_n}$ .

Then, we have:

$$\begin{aligned} \frac{a_1^2}{a_1 + a_2} + \frac{a_2^2}{a_1 + a_2} + \cdots + \frac{a_n^2}{a_n + a_1} &\geq \frac{(a_1 + a_2 + \cdots + a_n)^2}{(a_1 + a_2) + (a_2 + a_3) + \cdots + (a_n + a_1)} \\ &= \frac{(a_1 + a_2 + \cdots + a_n)^2}{2(a_1 + a_2 + \cdots + a_n)} \\ &= \frac{a_1 + a_2 + \cdots + a_n}{2} \end{aligned}$$