PROBLEM OF THE WEEK

Solution of Problem No. 4 (Spring 2009 Series)

**Problem:** The time–varying temperature of a body is given by a polynomial in time of degree $\leq 3$. Show that the average temperature of the body between 6:00 AM and 12:00 noon can be found by taking the average of the temperatures at two fixed times, $t_1$ and $t_2$, which are independent of which polynomial occurs. Also find $t_1$ and $t_2$. (Remark: the average of a function $f(x)$ over an interval $a \leq x \leq b$ is defined as $\frac{1}{b-a}\int\limits_a^b f(x)dx$.)

**Solution** (by Angel Plaza, ULPGC, Spain)

Under a suitable change of variable we can suppose the problem defined in the interval $[-1,+1]$. Let $P_3(x) = ax^3 + bx^2 + cx + d$ be the polynomial of degree $\leq 3$. Its average over the interval $[-1,+1]$ is then $\displaystyle\frac{1}{2}\int\limits_{-1}^{1} P_3(x)dx = \frac{1}{2}\left[\frac{ax^4}{4} + \frac{bx^3}{3} + \frac{cx^2}{2} + dx\right]_{-1}^{1} = \frac{b}{3} + d$.

In order to find $t_1$ and $t_2$ we set $t_2 = -t_1$ and therefore $\displaystyle\frac{1}{2}(P_3(t_1) + P_3(t_2)) = bt_1^2 + d$. Then, $bt_1^2 + d = b/3 + d$ if and only if $t_1 = \sqrt{\frac{1}{3}}$, and $t_2 = -\sqrt{\frac{1}{3}}$. The values for $t_1$ and $t_2$ in the given interval $[6, 12]$ are produced with the function $g(x) = 3x + 9$ which transforms $[-1,+1]$ into $[6,+12]$. So the solution is $t_1 = 9 + \sqrt{3}$ and $t_2 = 9 - \sqrt{3}$.

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